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Absolute Value Equations and Inequalities Guide Notes

The three types of open sentences that can involve absolute value:

$\lvert x\rvert = n$	$\lvert x\rvert < n$	$\lvert x\rvert > n$

A. **ABSOLUTE VALUE EQUATIONS:** $\lvert x\rvert = n$

When solving equations involving absolute value, we need to consider these cases

a. The value inside the absolute value symbols is **positive**.

$$x = n$$

b. The value inside the absolute value symbols is **negative**.

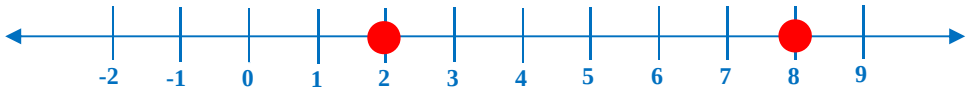
$$x = -n$$

If $\lvert x\rvert = n$, then $x = n$ or $x = -n$.

Sample Problem 1: Solve each equation and graph the solution set.

1. $\lvert x - 5\rvert = 3$

$x = 8$ *or* $x = 2$



$x - 5 = 3$
$x - 5 + 5 = 3 + 5$
$x = 8$

$x - 5 = -3$
$x - 5 + 5 = -3 + 5$
$x = 2$

Check:

$x = 8$	$x = 6$	$x = 2$
$\lvert 8 - 5\rvert = 3$	$\lvert 6 - 5\rvert = 3$	$\lvert 2 - 5\rvert = 3$
$\lvert 3\rvert = 3$	$\lvert 1\rvert = 3$	$\lvert -3\rvert = 3$
$3 = 3$	$1 \neq 3$	$3 = 3$

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Absolute Value Equations and Inequalities Guide Notes

2. $|3x + 4| = 17$

$x = \frac{13}{3}$ *or* $x = -7$

$3x + 4 = 17$

$3x + 4 - 4 = 17 - 4$

$3x = 13$

$\frac{3x}{3} = \frac{13}{3}$

$x = \frac{13}{3}$

$3x + 4 = -17$

$3x + 4 - 4 = -17 - 4$

$3x = -21$

$\frac{3x}{3} = \frac{-21}{3}$

$x = -7$

Check:

$x = \frac{13}{3}$	$x = -2$	$x = -7$
$\left\|3\left(\frac{13}{3}\right) + 4\right\| = 17$	$\|3(-2) + 4\| = 17$	$\|3(-7) + 4\| = 17$
$\|13 + 4\| = 17$	$\|-6 + 4\| = 17$	$\|-21 + 4\| = 17$
$\|17\| = 17$	$\|-2\| = 17$	$\|-17\| = 17$
$17 = 17$	$2 \neq 17$	$17 = 17$

3. $|2x - 5| - 9 = 6$

$x = 10$ *or* $x = -5$

$|2x - 5| - 9 + 9 = 6 + 9$

$|2x - 5| = 15$

$|2x - 5| = 15$

$2x - 5 = 15$

$2x - 5 + 5 = 15 + 5$

$2x = 20$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

$|2x - 5| = -15$

$2x - 5 = -15$

$2x - 5 + 5 = -15 + 5$

$2x = -10$

$\frac{2x}{2} = -\frac{10}{2}$

$x = -5$

Check:

$x = 10$	$x = 2$	$x = -5$
$\|2(10) - 5\| - 9 = 6$	$\|2(2) - 5\| - 9 = 6$	$\|2(-5) - 5\| - 9 = 6$
$\|20 - 5\| - 9 = 6$	$\|4 - 5\| - 9 = 6$	$\|-10 - 5\| - 9 = 6$
$\|15\| - 9 = 6$	$\|-1\| - 9 = 6$	$\|-15\| - 9 = 6$
$15 - 9 = 6$	$1 - 9 = 6$	$15 - 9 = 6$
$6 = 6$	$-8 \neq 6$	$6 = 6$

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Absolute Value Equations and Inequalities Guide Notes

B. **ABSOLUTE VALUE INEQUALITIES:** $|x| < n$

When solving inequalities of the form $|x| < n$, we need to find the **Intersection** of these cases:

a. The value inside the absolute value symbols is **less than the positive value of n**.

$$x < n$$

b. The value inside the absolute value symbols is **greater than the negative value of n**.

$$x > -n$$

If $|x| < n$, then $x < n$ and $x > -n$.

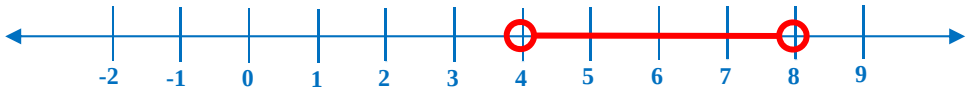
It also applies for $|x| \leq n$:

If $|x| \leq n$, then $x \leq n$ and $x \geq -n$.

Sample Problem 2: Solve each inequality and graph the solution set.

1. $|x - 6| < 2$

$4 < x < 8$



$x - 6 < 2$
$x - 6 + 6 < 2 + 6$
$x < 8$

$x - 6 > -2$
$x - 6 + 6 > -2 + 6$
$x > 4$

Check:

$x < 8$

$x = 7$	$x = 8$	$x = 9$
$\|7 - 6\| < 2$	$\|8 - 6\| < 2$	$\|9 - 6\| < 2$
$\|1\| < 2$	$\|8\| < 2$	$\|3\| < 2$
$1 < 2$	$2 \not< 2$	$3 \not< 2$

$x > 4$

$x = 3$	$x = 4$	$x = 5$
$\|3 - 6\| < 2$	$\|4 - 6\| < 2$	$\|5 - 6\| < 2$
$\|-3\| < 2$	$\|-4\| < 2$	$\|-1\| < 2$
$3 \not< 2$	$2 \not< 2$	$1 < 2$

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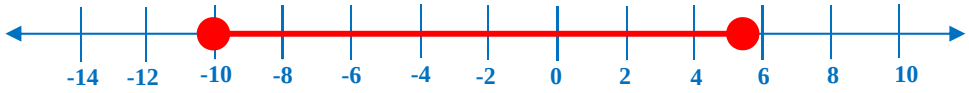
Absolute Value Equations and Inequalities Guide Notes

2. $|3x + 7| \leq 23$

$-10 \leq x \leq \frac{16}{3}$



$3x + 7 \leq 23$

$3x + 7 - 7 \leq 23 - 7$

$3x \leq 16$

$\frac{3x}{3} \leq \frac{16}{3}$

$x \leq \frac{16}{3}$ *or* $x \leq 5\frac{1}{3}$

$3x + 7 \geq -23$

$3x + 7 - 7 \geq -23 - 7$

$3x \geq -30$

$\frac{3x}{3} \geq \frac{-30}{3}$

$x \geq -10$

Check:

$x \leq \frac{16}{3}$ *or* $x \leq 5\frac{1}{3}$

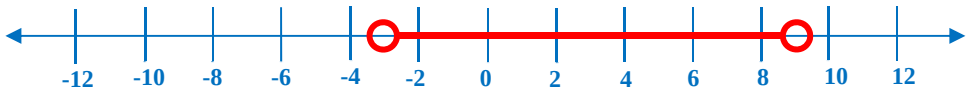
$x = 5$	$x = \frac{16}{3}$	$x = 6$
$\|3(5) + 7\| \leq 23$	$\left\|3\left(\frac{16}{3}\right) + 7\right\| \leq 23$	$\|3(6) + 7\| \leq 23$
$\|15 + 7\| \leq 23$	$\|16 + 7\| \leq 23$	$\|18 + 7\| \leq 23$
$\|22\| \leq 23$	$\|23\| \leq 23$	$\|25\| \leq 23$
$22 \leq 23$	$23 \leq 23$	$25 \not\leq 23$

$x \geq -10$

$x = -11$	$x = -10$	$x = -9$
$\|3(-11) + 7\| \leq 23$	$\|3(-10) + 7\| \leq 23$	$\|3(-9) + 7\| \leq 23$
$\|-33 + 7\| \leq 23$	$\|-30 + 7\| \leq 23$	$\|-27 + 7\| \leq 23$
$\|-26\| \leq 23$	$\|-23\| \leq 23$	$\|-20\| \leq 23$
$26 \not\leq 23$	$23 \leq 23$	$20 \leq 23$

3. $|3x - 9| - 4 < 14$

$-3 < x < 9$



$|3x - 9| - 4 + 4 < 14 + 4$

$|3x - 9| < 18$

$3x - 9 < 18$

$3x - 9 + 9 < 18 + 9$

$3x < 27$

$\frac{3x}{3} < \frac{27}{3}$

$x < 9$

$3x - 9 > -18$

$3x - 9 + 9 > -18 + 9$

$3x > -9$

$\frac{3x}{3} > \frac{-9}{3}$

$x > -3$

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Absolute Value Equations and Inequalities Guide Notes

Check: $x < 9$

$x = 8$	$x = 9$	$x = 10$
$\|3(8) - 9\| - 4 < 14$	$\|3(9) - 9\| - 4 < 14$	$\|3(10) - 9\| - 4 < 14$
$\|24 - 9\| - 4 < 14$	$\|27 - 9\| - 4 < 14$	$\|30 - 9\| - 4 < 14$
$\|15\| - 4 < 14$	$\|18\| - 4 < 14$	$\|21\| - 4 < 14$
$15 - 4 < 14$	$18 - 4 < 14$	$15 - 4 < 14$
$11 < 14$	$14 \not< 14$	$17 \not< 14$

$x > -3$

$x = -4$	$x = -3$	$x = -2$
$\|3(-4) - 9\| - 4 < 14$	$\|3(-3) - 9\| - 4 < 14$	$\|3(-2) - 9\| - 4 < 14$
$\|-12 - 9\| - 4 < 14$	$\|-9 - 9\| - 4 < 14$	$\|-6 - 9\| - 4 < 14$
$\|-21\| - 4 < 14$	$\|-18\| - 4 < 14$	$\|-15\| - 4 < 14$
$21 - 4 < 14$	$18 - 4 < 14$	$15 - 4 < 14$
$17 \not< 14$	$14 \not< 14$	$11 < 14$

C. **ABSOLUTE VALUE INEQUALITIES:** $|x| > n$

When solving inequalities of the form $|x| > n$, we need to find the union of these cases:

a. The value inside the absolute value symbols is **GREATER THAN THE POSITIVE VALUE OF** n.

$$x > n$$

b. The value inside the absolute value symbols is **LESS THAN THE NEGATIVE VALUE OF** n.

$$x < -n$$

If $|x| > n$, then $x > n$ or $x < -n$.

It also applies for $|x| \geq n$:

If $|x| \geq n$, then $x \geq n$ or $x \leq -n$.

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Absolute Value Equations and Inequalities Guide Notes

Sample Problem 3: Solve each equation and graph the solution set.

1. $|x+3| \geq 11$

$x+3 \geq 11$

$x+3-3 \geq 11-3$

$x \geq 8$

$x+3 \leq -11$

$x+3-3 \leq -11-3$

$x \leq -14$

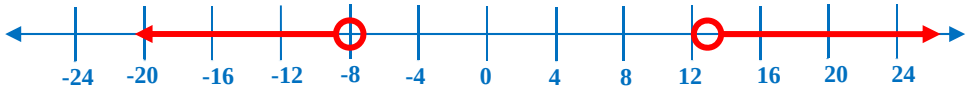
Check: $x \geq 8$

$x=7$	$x=8$	$x=9$
$\|7+3\| \geq 11$	$\|8+3\| \geq 11$	$\|9+3\| \geq 11$
$\|10\| \geq 11$	$\|11\| \geq 11$	$\|12\| \geq 11$
$10 \not\geq 11$	$11 \geq 11$	$12 \geq 11$

$x \leq -14$

$x=-13$	$x=-14$	$x=-15$
$\|-13+3\| \geq 11$	$\|-14+3\| \geq 11$	$\|-15+3\| \geq 11$
$\|-10\| \geq 11$	$\|-11\| \geq 11$	$\|-12\| \geq 11$
$10 \not\geq 11$	$11 \geq 11$	$12 \geq 11$

2. $|2x-5| > 21$



$2x-5 > 21$

$2x-5+5 > 21+5$

$2x > 26$

$\frac{2x}{2} > \frac{26}{2}$

$x > 13$

$2x-5 < -21$

$2x-5+5 < -21+5$

$2x < -16$

$\frac{2x}{2} < \frac{-16}{2}$

$x < -8$

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Absolute Value Equations and Inequalities Guide Notes

Check: $x > 13$

$x = 12$	$x = 13$	$x = 14$
$\|2(12) - 5\| > 21$	$\|2(13) - 5\| > 21$	$\|2(14) - 5\| > 21$
$\|24 - 5\| > 21$	$\|26 - 5\| > 21$	$\|28 - 5\| > 21$
$\|19\| > 21$	$\|21\| > 21$	$\|23\| > 21$
$19 \not> 21$	$21 \not> 21$	$23 > 21$

$x < -8$

$x = -7$	$x = -8$	$x = -9$
$\|2(-7) - 5\| > 21$	$\|2(-8) - 5\| > 21$	$\|2(-9) - 5\| > 21$
$\|-14 - 5\| > 21$	$\|-16 - 5\| > 21$	$\|-18 - 5\| > 21$
$\|-19\| > 21$	$\|-21\| > 21$	$\|-23\| > 21$
$19 \not> 21$	$21 \not> 21$	$23 > 21$

3. $|2x + 7| - 6 \geq 15$

$\|2x + 7\| - 6 + 6 \geq 15 + 6$	$2x + 7 \geq 21$	$2x + 7 \leq -21$
$\|2x + 7\| \geq 21$	$2x + 7 - 7 \geq 21 - 7$	$2x + 7 - 7 \leq -21 - 7$
	$2x \geq 14$	$2x \leq -28$
	$\frac{2x}{2} \geq \frac{14}{2}$	$\frac{2x}{2} \leq \frac{-28}{2}$
	$x \geq 7$	$x \leq -14$

Check: $x \geq 7$

$x = 6$	$x = 7$	$x = 8$
$\|2(6) + 7\| - 6 \geq 15$	$\|2(7) + 7\| - 6 \geq 15$	$\|2(8) + 7\| - 6 \geq 15$
$\|12 + 7\| - 6 \geq 15$	$\|14 + 7\| - 6 \geq 15$	$\|16 + 7\| - 6 \geq 15$
$\|19\| - 6 \geq 15$	$\|21\| - 6 \geq 15$	$\|23\| - 6 \geq 15$
$19 - 6 \geq 15$	$21 - 6 \geq 15$	$23 - 6 \geq 15$
$13 \not\geq 15$	$15 \geq 15$	$17 \geq 15$

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Absolute Value Equations and Inequalities Guide Notes

$x \leq -14$

$x = -13$	$x = -14$	$x = -15$
$\lvert 2(-13) + 7 \rvert - 6 \geq 15$	$\lvert 2(-14) + 7 \rvert - 6 \geq 15$	$\lvert 2(-15) + 7 \rvert - 6 \geq 15$
$\lvert -26 + 7 \rvert - 6 \geq 15$	$\lvert -28 + 7 \rvert - 6 \geq 15$	$\lvert -30 + 7 \rvert - 6 \geq 15$
$\lvert -19 \rvert - 6 \geq 15$	$\lvert -21 \rvert - 6 \geq 15$	$\lvert -23 \rvert - 6 \geq 15$
$19 - 6 \geq 15$	$21 - 6 \geq 15$	$23 - 6 \geq 15$
$13 \ngeq 15$	$15 \geq 15$	$17 \geq 15$

Sample Problem 4: The starting players of the school's varsity basketball team have an average scoring point between 9 and 20. Write an absolute value inequality describing the average scoring point of the starting players.

Step 1 Write the inequality. $9 < x < 20$

Step 2 Determine the midpoint. $\frac{9 + 20}{2} = \frac{29}{2} = 14.5$

Step 3 Determine the distance from the midpoint. $\lvert 20 - 14.5 \rvert < 5.5$ $\lvert 9 - 14.5 \rvert < 5.5$

Step 4 Write the absolute value inequality. Make sure that the inequality symbol is correct.

$$\lvert x - \textit{midpoint} \rvert < \textit{distance from the midpoint}$$

$$\lvert x - 14.5 \rvert < 5.5$$